STAT/MA 41600
In-Class Problem Set #8: September 7, 2018

1. Consider five 4-sided dice that are each painted, without numbers:
One has all 4 sides painted blue;
one has 3 sides painted blue and 1 side painted white;
one has 2 sides painted blue and 2 sides painted white;
one has 1 side painted blue and 3 sides painted white;
one has all 4 sides painted white.

A die is selected at random (with all 5 dice equally likely). Let $X = 1$ if the result is blue, and $X = 0$ otherwise (i.e., if the result is white). What is the probability mass function of X?

2. Consider the five regular kinds dice that come in the shapes of Platonic solids, namely: a 4-sided die (tetrahedron), a 6-sided die (cube), an 8-sided die (octahedron), a 12-sided die (dodecahedron), and a 20-sided die (icosahedron). Audrey has one die of each type. She selects one of the dice at random from her bag of dice (all are equally likely to be chosen). Let X denote the value that appears on the selected die. What is the probability mass function of X?

3. Rafael repeatedly selects cards from the deck, according to the following scheme: If he chooses a Jack, Queen, or King, then he removes it from the deck, along with all of the other Jacks, Queens, and Kings. Otherwise, he just puts the selected card back into the deck, without modifying the deck.

Rafael continues this process until he gets the first Jack, Queen, or King, and then he stops afterward.

Let X denote the number of draws that Rafael makes altogether.

3a. Find the probability mass function of X.

3b. Verify that the values of the probability mass function of X sum to 1.

4. Consider a collection of 6 bears. There is a pair of red bears consisting of one father bear and one mother bear. There is a similar green bear pair, and a similar blue bear pair. These 6 bears are all placed in a straight line, and all arrangements in such a line are equally likely. A bear pair is happy if it is sitting together. Let X denote the number of happy bear pairs. Find the probability mass function of X.

(There are three pairs altogether, so we know that $0 \leq X \leq 3$.)