STAT/MA 41600
In-Class Problem Set #7: September 5, 2018
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Problem Set 7 Answers

1a. We see that X is a continuous random variable.
1b. Since Y can only take on the values 0, 1, 2, 3, 4, then Y is a discrete random variable.
1c. No, (X, Z) is not a random variable. It is a pair of random variables. A random variable maps outcomes (elements of the sample space) to $\mathbb{R}$, i.e., to real numbers.
1d. We see that $X + Z$ is a continuous random variable; it is the sum of two measurements.

2a. We have $P(X = 0) = (4/6)(3/5)(2/4) = 1/5$.

2b. We have $P(X = 1) = (2/6)(4/6)(3/5) + (4/6)(2/5)(3/5) + (4/6)(3/5)(2/4) = 37/75$.

2c. We have $P(X = 2) = (2/6)^2(4/6) + (2/6)(4/6)(2/5) + (4/6)(2/5)(2/5) = 182/675$.

2d. We have $P(X = 3) = (2/6)^3 = 1/27$.

3a. We note that $X \geq 3$ if and only if he does not select a Jack, Queen, or King on the first two moves, so $P(X \geq 3) = (40/52)^2$.

3b. Similarly, we have $X \geq 10$ if and only if he does not select a Jack, Queen, or King on the first nine moves, so $P(X \geq 10) = (40/52)^9$.

3c. Similarly, we have $X \geq 100$ if and only if he does not select a Jack, Queen, or King on the first ninety-nine moves, so $P(X \geq 100) = (40/52)^{99}$.

3d. Finally, we have $X \geq n$ if and only if he does not select a Jack, Queen, or King on the first $n - 1$ moves, so $P(X \geq n) = (40/52)^{n-1}$.

4. We go in reserve order.
4d. We have $X = 4$ if and only if all of the dice are 4's, so $P(X = 4) = (1/4)^3 = 1/64$.
4c. We have $X = 3$ if and only if all of the dice are 3's or 4's, but they are not all 4's, so $P(X = 3) = (2/4)^3 - (1/4)^3 = 7/64$.
4b. We have $X = 2$ if and only if all of the dice are 2's or 3's or 4's, but they are not all 3's or 4's, so $P(X = 2) = (3/4)^3 - (2/4)^3 = 19/64$.
4a. We have $X = 1$ if and only if all of the dice are 1's or 2's or 3's or 4's, but they are not all 2's or 3's or 4's, so $P(X = 1) = (4/4)^3 - (3/4)^3 = 37/64$.