STAT/MA 41600
In-Class Problem Set #5: August 31, 2018

1. Suppose that 37% of the pages of an analysis textbook have a theorem, 29% of the pages of an algebra textbook have a theorem, and 20% of the pages of a probability textbook have a theorem. The academic advisors have encouraged students to take at most one math course at a time, and as a result, 5% of students are taking analysis, 13% are taking algebra, 10% are taking probability, and the other 72% are not taking a math course.

If you randomly walk up to a table at the local coffee shop and peek over the student's shoulder, and the student happens to be reading a theorem in a math book, what is the probability that student is enrolled in a probability course?

2. Consider a collection of six bears of different colors. Mary likes purple and orange bears. When she reaches into her collection and finds a purple or orange bear, she gets happy and runs to her room, and grabs 2 more bears of the *same color*, and puts all 3 bears back into the collection. On the other hand, if she finds a bear that is not purple or orange, she throws it away forever.

2a. Suppose Mary does this procedure 1 time before Dean arrives. If Dean randomly selects a purple bear, what is the conditional probability that Mary was unhappy (i.e., that she threw away a bear)?

2b. Suppose (instead) that Mary does this procedure 2 times before Dean arrives. If Dean randomly selects a purple bear, what is the conditional probability that Mary was unhappy both times (i.e., that she threw away two bears)?

3. Rafael and Susan are creating a new card game. Rafael first selects a card from the deck. If he chooses a Jack, Queen, or King, then he removes it from the deck, along with all of the other Jacks, Queens, and Kings. Otherwise, he just puts the selected card back into the deck, without modifying the deck.

3a. Suppose Susan checks the deck after Rafael does the procedure given above, and she finds an "Ace" card. What is the probability that Rafael has not yet removed any cards from the deck?

3b. Suppose that (instead) Rafael does the procedure three times during the long holiday weekend, with the same deck, and he does not reset the deck afterwards. (In other words, if he removes cards anytime during the weekend, they are permanently removed for the rest of the weekend.) Suppose Susan checks the deck after the weekend is over, and she finds an "Ace" card. What is the probability that Rafael has not yet removed any cards from the deck?

4. Bob rolls a 6-sided die until the first value of "2" appears for the first time, and then he stops afterwards. Suppose that it takes him n rolls.

Based on that number n, Bob's friend Alice flips a coin exactly n times. What is the probability that Alice gets no heads at all, during her n flips?