STAT/MA 41600
In-Class Problem Set #2: August 24, 2018
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Problem Set 2 Answers

1a. Method #1: There are $5! = 120$ equally likely outcomes, and from 1b in the previous problem set, the daughter bears sit next to each other in exactly 48 of these outcomes, so the probability is $48/120 = 2/5$. Method #2: The daughters have $\binom{5}{2} = 10$ equally likely pairs of seats where they can sit (without worrying about which daughter is in which place), and they are sitting next to each other in exactly 4 of these 10 ways, so the probability is $4/10 = 2/5$.
1b. Method #1: There are $5! = 120$ equally likely outcomes, and from 1c in the previous problem set, the daughter bears sit next to each other and the son bears sit next to each other in exactly 24 of these outcomes, so the probability is $24/120 = 1/5$. Method #2: The daughters have $\binom{5}{2} = 10$ equally likely pairs of seats where they can sit (without worrying about which daughter is in which place), and they are sitting next to each other, and leaving room for the sons to sit next to each other too, in exactly 2 of these 10 ways, so the probability is $2/10 = 1/5$.
1c. Method #1: There are $5! = 120$ equally likely outcomes. If none of the son bears are sitting next to each other, then they must be staggered, so their seats are known, and this forces the seats for the daughter bears are known too, so none of the son bears are sitting next to each other in $(2!)(3!) = (2)(6) = 12$ ways, so the probability is $12/120 = 1/10$. Method #2: The seats next to the daddy bear are occupied by sons with probability $(3/5)(2/4) = 3/10$, and given that this happens, the seat across from the daddy bear is occupied by a son with probability $1/3$, so the desired probability is $(3/10)(1/3) = 1/10$.

2a. There are $6^3 = 216$ equally likely outcomes altogether, and as in question 2c in the previous problem set, exactly 91 such outcomes have at least one purple. So the probability is $91/216$.
2b. Yes, every outcome has either 1, 2, or 3 colors altogether, since we are picking three bears, so $C_1 \cup C_2 \cup C_3$ contains every outcome in the sample space. Moreover, the C_j's are disjoint since an outcome cannot have more than one tally of the colors, so it cannot be in more than one of the C_j's. So C_1, C_2, C_3 form a partition of the sample space.
2c. We have $P(C_1) = (6)(1/6)^3 = 1/36$ and $P(C_2) = (3)(6)(5)(1/6)^3 = 5/12$ and $P(C_3) = (6)(5)(4)(1/6)^3 = 5/9$. As a reality check, we have $P(C_1)+P(C_2)+P(C_3) = 1/36+5/12+5/9 = 1$.

3a. The A_j's are not disjoint so they do not form a partition of the sample space.
3b. As in equation 3b from the previous problem set, there are 8 outcomes in $A_1 \cap A_2 \cap A_3$, and all 6^3 outcomes are equally likely, so we get $P(A_1 \cap A_2 \cap A_3) = 8/6^3 = 1/27$.
3c. As in equation 3c from the previous problem set, there are 152 outcomes in $A_1 \cup A_2 \cup A_3$, and all 6^3 outcomes are equally likely, so we get $P(A_1 \cup A_2 \cup A_3) = 152/6^3 = 19/27$.

4. She selects no yellow bears during the process if there is a value $j \geq 0$, such that she selects exactly j bears that are not purple, orange, or yellow, and then on the $(j+1)$st draw she selects purple or orange. So the desired probability is $\sum_{j=0}^{\infty}(3/6)^j(2/6) = \frac{2/6}{1-3/6} = 2/3$.

Method #2: Essentially only the purple, orange, and yellow bears affect the probability of whether or not she selects any yellow bears during this process. To see this, consider the first draw that is either purple, orange, or yellow. If it is purple or orange, then she does not select any yellow bears in this process. If it is yellow, then she does select at least one yellow bear in this process. The first draw that is either purple, orange, or yellow has a probability of $2/3$ of being purple or orange. So the desired probability is $2/3$.