STAT/MA 41600
In-Class Problem Set #1: August 22, 2018

1. Suppose 6 bears (a daddy bear and two daughter bears and three son bears) sit around a circular campfire. The daddy bear always sits in the same place, but the locations of the other five bears are random.
1a. How many outcomes are there for the bears to sit?
1b. In how many of these outcomes do the two daughter bears sit next to each other?
1c. In how many of these outcomes do the two daughter bears sit next to each other *and* (simultaneously) the three son bears sit next to each other?

2. Consider a bear family with 6 bears that have 6 different colors. Pick three of the bears, with replacement in between the selections. Keep track of the bears selected, in order.
2a. How many outcomes are there in the sample space?
2b. How many possible events are there?
2c. Let B denote the event that at least one purple bear is selected among the three bears chosen. How many outcomes are in event B? Hint: Think about the complement event.

3. Same setup as question #2. In this setup, suppose that Mary loves purple and orange bears. Let A_j denote the event that her jth selection was purple or orange.
3a. How many outcomes are in the event A_1?
3b. How many outcomes are in the event $A_1 \cap A_2 \cap A_3$?
3c. How many outcomes are in the event $A_1 \cup A_2 \cup A_3$? (Hint: For this part, it might be easier to calculate the number of outcomes in the complement event, i.e., in $(A_1 \cup A_2 \cup A_3)^c$, which is equal to $A_1^c \cap A_2^c \cap A_3^c$.)

4. Calculus review.
4a. Find the value of $\sum_{j=0}^{\infty} \frac{2^j}{j!}$ and the value of $\sum_{j=3}^{\infty} \frac{2^j}{j!}$.
4b. Find the value of $\sum_{j=1}^{\infty}(1/3)^{j-1}(2/3)$ and the value of $\sum_{j=4}^{\infty}(1/3)^{j-1}(2/3)$.

Extra brain stretch for tonight. (Does not need to be completed in class today.)
Is $\sum_{j=1}^{\infty} 1/j$ a convergent series? Why or why not? Hint: Think about the natural logarithm function.
Is $\sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j}$ a convergent series? Why or why not? Hint: $\ln(1+x) = \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j} x^j$ for $-1 < x \leq 1$.